Statistics

Lecture 8

Feb 19-8:47 AM

SG 14

Data
- 1) Qualitative
- 2) Quantitative
  - 1) Discrete
  - 2) Continuous

Let x be a discrete random variable with Prob. dist. $P(x)$.

It gives us the prob. of all possible outcomes.

1) Table or chart
2) Graph
3) Formula
4) use of def. of prob.

Oct 20-5:09 PM

Consider the chart below

x	$P(x)$
1	.2
2	.5
3	.3

1) verify $\sum P(x) = 1$ ✓

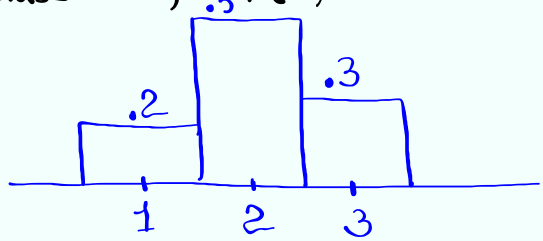
$.2 + .5 + .3 = 1$

2) find $P(x \geq 2)$

$.5 + .3 = \boxed{.8}$

3) Draw Prob. dist. histogram.

$x \to$ class MP , $P(x) \to$ Rel.F.



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Some rules

1) $0 \leq P(x) \leq 1$

2) $\sum P(x) = 1$

3) $P(x) = 1 \leftrightarrow$ Sure event

4) $P(x) = 0 \leftrightarrow$ Impossible event

5) $0 < P(x) \leq .05 \leftrightarrow$ Rare event

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Consider the Chart below

x	$P(x)$
1	.1
2	.3
3	.4
4	.2

1) Find $P(X=4)$.

$= 1 - (.1 + .3 + .4)$

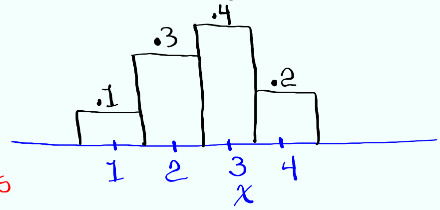
Total Prob.

$= 1 - .8 = \boxed{.2}$

2) Find $P(2 \le x \le 3) =$

$.3 + .4 = \boxed{.7}$

3) Draw Prob. dist. histogram.



Clear all lists

$x \to L1$, $P(x) \to L2$

use $\boxed{\text{1-Var Stats}}$

with L1 & L2

$\bar{x} = 2.7$

$S = S_x =$ blank

$n = 1$ ← Total Prob.

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A piggy bank has 3 nickels & 2 dimes.

Take 2 Coins with replacement.

NN ND DN DD

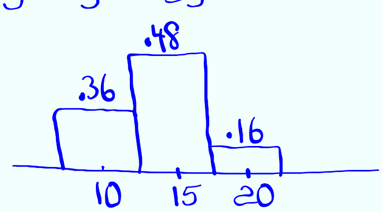
10¢ 15¢ 20¢

$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = \boxed{.36}$

$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{25} = \boxed{.48}$

$P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = \boxed{.16}$

¢	P(¢)
10	.36
15	.48
20	.16



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¢	P(¢)
10	.36
15	.48
20	.16

L1 (¢ column), L2 (P(¢) column)

¢ → L1

P(¢) → L2

use $\boxed{\text{1-Var Stats}}$

with L1 & L2

$\bar{x} = 14$

$S = S_x = \text{blank}$

$n = 1$

Total Prob.

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Working with x & $P(x)$

$\boxed{\text{Mean} \quad \mu = \sum x\,p(x)}$

mu

$\boxed{\text{Variance} \quad \sigma^2 = \sum x^2 p(x) - \mu^2}$

Sigma

$\boxed{\text{Standard deviation} \quad \sigma = \sqrt{\sigma^2}}$

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Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) $\sum P(x) = 1$ ✓

2) $\sum xp(x) = 1.9$

3) $\sum x^2p(x) = 4.1$

4) $\mu = \sum xp(x) = \boxed{1.9}$

5) $\sigma^2 = \sum x^2p(x) - \mu^2 = 4.1 - 1.9^2 = \boxed{.49}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{.49} = \boxed{.7}$

68% Range $\mu \pm \sigma = 1.9 \pm .7 \rightarrow \boxed{1.2 \text{ to } 2.6}$

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Using TI to find μ & σ:

$x \rightarrow L1$, $P(x) \rightarrow L2$

L1	L2
1	.3
2	.5
3	.2

Now use $\boxed{\text{1-Var Stats}}$ with L1 & L2.

$\mu = \bar{x} = 1.9$

$\sigma = \sigma_x = .7$

$n = 1$ ← Total prob.

How to find σ^2:

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4{:}\,\sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

$\sigma^2 = .49 = \frac{49}{100}$

Reduced Fraction $\boxed{\text{Math}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

Oct 20-5:52 PM

3 nickels, 2 dimes

Take 2 Coins without replacement

NN — 10¢

ND, DN — 15¢

DD — 20¢

$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10} = \boxed{.3}$

$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20} = \frac{6}{10} = \boxed{.6}$

$P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10} = \boxed{.1}$

¢	P(¢)
10	.3
15	.6
20	.1

¢ → L1, P(¢) → L2

use 1-Var Stats with L1 & L2

$\mu = \bar{x} = \boxed{14}$ $\sigma = \sigma_x = \boxed{3}$ $\sigma^2 = \boxed{9}$

VARS 5:Statistics 4:σ_x x^2 Enter

usual Range (95% Range)

$\mu \pm 2\sigma = 14 \pm 2(3) \rightarrow \boxed{8 \text{ To } 20}$

SG 14 & Part of SG 15

Oct 20-5:58 PM

Application:

Expected Value $\mu = \bar{x}$

Ex: I Sold 25 tickets for \$10 each. I took one ticket randomly as the winning tkt, owner gets a Calculator worth \$100.

Expected Value per ticket Sold.

Net	P(Net)	
10 − 100	1/25	winning tkt
10 − 0	24/25	$\overline{\text{winning tkt}}$

Net → L1, P(Net) → L2

use 1-Var Stats with L1 & L2.

$E.V. = \mu = \bar{x} = \boxed{6}$

find σ^2.

VARS 5:Statistics 4:σ_x x^2 Enter

$\sigma^2 = 384$

Oct 20-6:18 PM

You buy an insurance policy for $100 for your luggage.

Policy pays $1000 if any damages.

Prob. of any damage is .5%.

Find expected value per policy sold.

Net	P(Net)	
100 − 1000	.5% = .005	damage
100 − 0	.995	~~damage~~

Net → L1

P(Net) → L2

E.V. = $\mu = \bar{x}$

$\boxed{95}$

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2

Find σ^2 → $\boxed{4975}$

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Pay me $5 and draw a card from a full deck.

If you draw	I give you
Ace	$50
Face	$ 5
Any other card	nothing

Find E.V. per bet for the house.

Net	P(Net)	
5 − 50	4/52	Ace
5 − 5	12/52	Face
5 − 0	36/52	Any other card

Net → L1

P(Net) → L2

E.V. = $\mu = \bar{x}$

$\boxed{0}$

SG 14 & 15

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Binomial Prob. Dist. (SG 16)

1) n independent Events

2) only two outcomes per event

$P(\text{Success}) = p \quad P(\text{Failure}) = q$

$p + q = 1$

$q = 1 - p$

3) p & q do not change for all n events.

4) $x \to$ # of Successes

$n - x \to$ # of failures

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

${}_nC_x \to$ # of combinations that x Successes happens.

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Consider a binomial prob. dist. with $n = 8$ and $p = .6$.

Find $P(x = 3) = {}_8C_3 \cdot (.6)^3 \cdot (.4)^5$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 56 \cdot (.6)^3 (.4)^5$

$n - x = 8 - 3 = 5$

$q = 1 - p = 1 - .6 = .4$

$= \boxed{.124}$

8 [Math] [→] PRB [↓] [nCr] 3 [Enter]

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I flip a fair coin 10 times,
find the prob. that we get 6 tails.

$n=10$ $\quad p=.5,\ q=.5$ $\quad x=6$

$n-x=4$

$P(x=6) = {}_{10}C_6 \cdot (.5)^6 \cdot (.5)^4$

$= 210 \cdot (.5)^6 \cdot (.5)^4$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= \boxed{.205}$

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You are taking a multiple-choice quiz with 12 questions. $n=12$

Each question has 4 choices with only one correct choice. $p=\frac{1}{4}=.25$ $\quad q=\frac{3}{4}=.75$

You are making random guesses.

P(Guess exactly 5 Correct Ans)

$P(x=5) = {}_{12}C_5 \cdot (.25)^5 \cdot (.75)^7$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$ $\quad = 792 \cdot (.25)^5 \cdot (.75)^7$

$12-5=7$ $\quad = \boxed{.103}$

P(Guess all Correct)

$P(x=12) = {}_{12}C_{12} \cdot (.25)^{12} \cdot (.75)^0$

$= \boxed{5.96 \times 10^{-8}}$

SG 14 & 15 Started SG 16

Calc. Matters

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